Rate-Optimum Beamforming Transmission in MIMO Rician Fading Channels

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Abstract

In this doctoral thesis, the focus is on the capability of MIMO systems to increase channel capacity. The capacity achieved by MIMO systems is closely related to the "channel knowledge" model which is assumed at both ends of the MIMO link. Considering the case of MIMO complex Gaussian ergodic channels, with perfect Channel State Information at the receiver and Channel Distribution Information at the transmitter, the "ergodic capacity" is the maximum average mutual information between transmitter and receiver and is achieved by a unique optimum spatial precoding transmission. For the case of beamforming transmission, the maximum average mutual information is achieved by the "optimum beamformer" and is referred to as "ergodic beamforming capacity". Considering spatially correlated MIMO Rician flat fading channels, there is no closed-form expression for the optimum beamformer. In this case, its calculation is performed numerically and is very complex for real time applications. In this work, it is proven that the aforementioned complex, multi-dimensional, convex constrained optimization problem can be transformed to an 1-D optimization problem, which can be solved very fast using standard 1-D algorithms. This proof was based on geometrical properties, basis transformations and the Karush-Kuhn-Tucker (KKT) conditions. Simulations demonstrate that the proposed 1-D method has significantly lower computational complexity compared to multi-dimensional algorithms and that in some operational environments the ergodic beamforming capacity is very close or equal to the ergodic capacity. Additionally, the 3GPP MIMO channel model is employed in order to study further (via simulations) the performance of the optimum beamformer in practical operational scenarios (urban micro/macro-cellular with/without LOS component and suburban macro-cellular environments). Simulations demonstrate that the optimum beamformer shows high performance in all cases, a fact that

*Doctoral Thesis Advisor: Serafim Karaboyas, Assistant Professor justifies the significance of the proposed solutions and the contribution of this work.

1 Introduction

1.1 Ergodic capacity

Multiple-Input Multiple-Output (MIMO) systems employ multiple transmit and receive antennas and exploit the fluctuations in the (received) signal level due to multipath propagation (multipath fading) in order to increase spectral efficiency, improve the Quality of Service and coverage and mitigate interference. These benefits are achieved without the expense of additional bandwidth and make MIMO a very attractive and promising option for future mobile communication systems, especially when combined with the benefits of orthogonal frequency-division multiplexing (OFDM). The most important techniques employed by MIMO systems in order to achieve the aforementioned benefits are beamforming, diversity and spatial multiplexing. In this doctoral thesis, the focus is on the capability of MIMO systems to increase spectral efficiency. A MIMO system can achieve much higher channel capacity than a conventional Single-Input Single-Output (SISO) system, and it can be proven that the achieved capacity increases linearly with the number of transmit or receive antenna elements [1]. However, the capacity achieved by MIMO systems is closely related to the channel knowledge model which is assumed at both ends of the link. Assuming perfect channel knowledge, referred to as perfect Channel State Information (CSI), at both ends of the link (transmitter-receiver), the spatial pre-coding transmission scheme that achieves capacity was presented in [1]-[2], and includes transmission along the right singular vectors of the channel matrix combined with "water-filling" for optimum power allocation between the transmit directions. However, perfect CSI at the transmitter is practically unrealistic, mainly due to the inevitable delay in the control channel which is used to feed back the CSI from the receiver or due to the delay in the channel estimation algorithm employed at the transmitter. Instead, it is more realistic and practical to assume that the transmitter has knowledge of the parameters of the MIMO channel distribution, since the channel statistics usually remain invariant in a large time window, (tens to hundreds of times larger than the coherence time). This channel knowledge model is referred to as Transmitter Channel Distribution Information (CDIT) [3]. In a CDIT model the rate-optimum transmission maximizes the average mutual information between transmitter and receiver and the maximum rate achieved in this case is referred to as "ergodic capacity". Considering MIMO complex Gaussian ergodic channels with perfect CSI at the receiver and CDIT, the optimum spatial pre-coding transmission has been addressed in the literature (i.e. methods for the calculation of the optimum transmit covariance matrix have been proposed) for the following channel models [4]-[5]:

a. MIMO Rayleigh flat fading channels. This CDIT model is referred to as Channel Covariance Information (CCI).

b. Spatially uncorrelated MIMO Rician flat fading channels with a unit covariance matrix. This CDIT model is referred to as Channel Mean Information (CMI).

c. Spatially correlated or uncorrelated with a non-unit covariance matrix MIMO Rician flat fading channels. This CDIT model is referred to as combined CMI-CCI model.

1.2 Ergodic beamforming capacity

In MIMO systems, when the transmit covariance matrix is constrained to be rank-1, then all the available power is transmitted along a unique direction with the help of a beamforming vector, as it is shown in Figure 1.

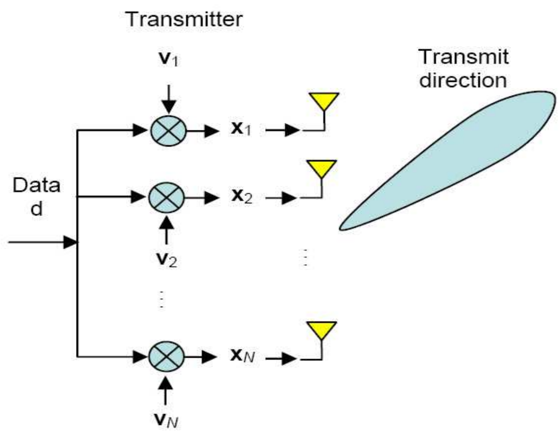

Figure 1: Beamforming transmission. $\mathbf{v} = [\mathbf{v}_1, \mathbf{v}_2, \dots \mathbf{v}_N]$ is the beamforming vector and $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_N] = d\mathbf{v}$ is the transmit signal vector.

The beamforming vector that maximizes the average mutual information for this constrained scenario is referred to as "optimum beamformer" and the achieved average mutual information as "ergodic beamforming capacity". There are several reasons why it is important to consider the optimum beamforming transmission in MIMO systems:

a. The complexity of the system and as a consequence the overall cost are significantly reduced.

b. There are operational scenarios (i.e. channels) where the ergodic beamforming capacity is very close to the ergodic capacity, which is achieved by higher rank transmission schemes.

c. The ergodic beamforming capacity does not coincide with the ergodic capacity of the channel, however, this is possible when a specific necessary and sufficient condition is satisfied by the channel distribution. This condition is expressed by a mathematical inequality and is referred to in the literature as the "optimality of beamforming condition" [6].

The solution of the optimum beamforming problem has been addressed extensively in the literature for the CCI and CMI models. For these two cases, closed-form solutions have been derived: the optimum beamformer coincides with the dominant eigenvector of the channel correlation matrix. However, the corresponding solution for the combined CMI-CCI model has received less attention. For this CDIT model, there is no closed-form expression for the optimum beamformer and hence, the solution of the related optimization problem remains very complex for real time applications.

In this work, it is proven that the aforementioned complex, multi-dimensional, convex constrained optimization problem for the combined CMI-CCI model can be transformed to a simple and equivalent 1-D optimization problem, which can be solved very fast using standard 1-D algorithms (gradient-based or direct search methods). This proof was based on geometrical properties of complex vector spaces, basis transformations and the Karush-Kuhn-Tucker (KKT) conditions. Moreover, a special (simpler) solution is proven for MIMO $2 \times M$ and special cases of MIMO $N \times M$ systems (e.g. MIMO systems with rank deficient transmit covariance matrix), where N/M is the number of transmit/receive antenna elements, respectively. The proof of this special case was based on a geometric approach, where the definition of the external product between vectors in high-dimensional complex vector spaces was exploited.

2 Rate-Optimum Beamforming Transmission in MIMO Rician flat fading channels

2.1 MISO systems

We consider a MISO $N \times 1$ flat fading channel with a complex Gaussian distribution $\mathbf{h} \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{R})$, with mean $\boldsymbol{\mu} \neq \mathbf{0}$ and covariance $\mathbf{R} \neq \mathbf{I}_N$ (i.e. a spatially correlated or uncorrelated with a non-unit covariance matrix MISO flat fading channel is assumed, with a Rician distribution for the amplitude of the elements of the channel vector $\mathbf{h}$). Assuming perfect CSI at the receiver and CDIT, the rate-optimum beamforming transmission for the channel model under consideration is the solution of an 1-D optimization problem, which is expressed by the following theorem [7]-[8]:

Theorem 1 The optimum beamformer $\mathbf{v}_{\text{opt}}$, for a MISO Rician flat fading channel with N transmit antenna elements ($N \geqslant 2$), mean value $\boldsymbol{\mu}$ ($\boldsymbol{\mu} \in \mathbb{C}^{1\times N}$, $\boldsymbol{\mu} \neq \mathbf{0}$) and transmit covariance matrix $\mathbf{R}$ ($\mathbf{R} \in \mathbb{H}_+^N$, $\mathbf{R} \neq \mathbf{I}_N$), can be calculated from the following 1-D optimization problem:

$$\mathbf{v}_{\text{opt}} = arg \max_{\mathbf{v} \in \mathbf{S}_o} \mathcal{I}_{\text{bf,avg}}(\text{SNR}, \mathbf{v}) \tag{1}$$

$$\mathbf{S}_o = \{\mathbf{v}_\theta \,; \theta \in [0, \phi]\} \tag{2}$$

where:

a. The average mutual information in (1) is expressed as:

$$\mathcal{I}_{\mathrm{bf,avg}}\left(\mathrm{SNR}, \mathbf{v}\right) = \mathcal{E}_{\mathbf{h}}\left[\log_2 \det\left(\mathbf{I}_M + \mathrm{SNR}\mathbf{h}\mathbf{v}^{\dagger}\mathbf{v}\mathbf{h}^{\dagger}\right)\right] \quad (3)$$

b. ϕ is the angle between $\boldsymbol{\mu}$ and the complex conjugate transpose of the dominant eigenvector of the channel transmit covariance matrix $\mathbf{R}$, denoted as $\mathbf{U}_{\bullet 1}^{\dagger}$, i.e.

$$\phi = \cos^{-1}\left(|\mathbf{m}\mathbf{U}_{\bullet 1}|\right) \quad (4)$$

with $\mathbf{m} = \boldsymbol{\mu}/\|\boldsymbol{\mu}\|_2$

c. $\mathbf{v}_{\theta}$ in (2) is expressed as:

$$\mathbf{v}_{\theta} = \cos\theta\left[1\ \mathbf{Z}(r_{\theta}\mathbf{I}_{N-1} - \mathbf{G})^{-1}\right]\mathbf{W}^T\mathbf{U}^{\dagger} \quad (5)$$

where:

i. $\mathbf{U}$ is the eigenvector matrix of $\mathbf{R}$ and $\mathbf{W}$ is a complex $N \times N$ orthonormal matrix with its first column defined as $\mathbf{W}_{\bullet 1} = \mathbf{U}^T\mathbf{m}^T$, whereas the rest of its columns ($\mathbf{W}_{\bullet i}$, $i = 2, ..., N$) are arbitrarily chosen, with the restriction that $\mathbf{W}^{\dagger}\mathbf{W} = \mathbf{I}_N$. Moreover, $\mathbf{G}$ and $\mathbf{Z}$ are defined as:

$$\mathbf{G} = \begin{pmatrix} \mathbf{K}_{22} & \cdots & \mathbf{K}_{2N} \\ \vdots & \ddots & \vdots \\ \mathbf{K}_{N2} & \cdots & \mathbf{K}_{NN} \end{pmatrix} \quad (6)$$

$$\mathbf{Z} = \left[\mathbf{K}_{12}\ \mathbf{K}_{13}\ \cdots\ \mathbf{K}_{1N}\right] \quad (7)$$

where $\mathbf{K}_{lm}$ is the l^{th} row and m^{th} column element of matrix $\mathbf{K}$, defined as:

$$\mathbf{K} = \sum_{i=1}^{N} \lambda_i(\mathbf{R})\mathbf{W}_{i\bullet}^T\mathbf{W}_{i\bullet}^* \quad (8)$$

with $\lambda_i(\mathbf{R})$ the i^{th} eigenvalue of $\mathbf{R}$.

ii. r_{θ} is the maximum real root of the $2(N-1)$-degree polynomial:

$$P(x;\theta) = \cos^2\theta \sum_{i=1}^{N-1} |\mathbf{Z}\mathbf{g}_i|^2 \left[\prod_{\substack{j=1 \\ j\neq i}}^{N-1} (x - \lambda_i(\mathbf{G}))^2\right] - \sin^2\theta \prod_{i=1}^{N-1} (x - \lambda_i(\mathbf{G}))^2 \quad (9)$$

where $\mathbf{g}_i \in \mathbb{C}^{(N-1)\times 1}$ is the i^{th} eigenvector of matrix $\mathbf{G}$.

Theorem 1 implies that the optimum beamformer belongs to a continuous trajectory (the continuity of the trajectory can be mathematically proven) that is defined by the vectors of $\mathbf{S}_o$ (see (2)), which lies on the surface of the unit-radius Euclidean ball, starts from $\mathbf{m}$ (for $\theta = 0$) and ends to $\mathbf{U}_{\bullet 1}^{\dagger}$ (for $\theta = \phi$). This is visualized in Figure 2.

Theorem 2 below ([7]-[8]) provides an alternative geometrically-based approach, especially for MISO systems with $N = 2$ transmit antenna elements.

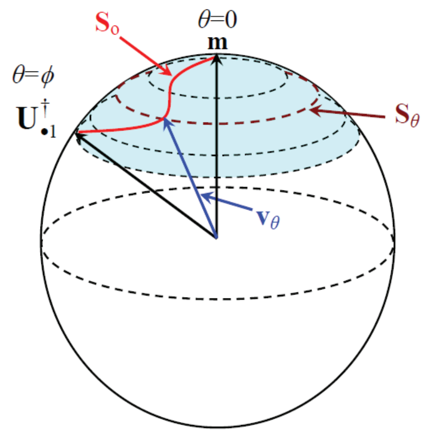

Figure 2: Geometric interpretation of 1-D method (Theorem 1).

Moreover, this theorem is also mathematically valid for the following special cases, with $N > 2$:

a. When $\boldsymbol{\mu}$ is a point in the hyperplane defined by $\mathbf{U}^{\dagger}_{\bullet 1}$ and $\mathbf{U}^{\dagger}_{\bullet 2}$ (i.e. the two dominant eigenvectors of $\mathbf{R}$).

b. When the channel covariance matrix has two eigenvalues, $\lambda_1(\mathbf{R})$ and $\lambda_2(\mathbf{R})$ ($\lambda_1(\mathbf{R}) \geqslant \lambda_2(\mathbf{R})$) with algebraic multiplicity one and $N-1$, respectively, or it is rank deficient, with $rank\{\mathbf{R}\} \leqslant 2$.

Theorem 2 For MISO systems with $N = 2$, $\mathbf{v}_\theta$ can be expressed by the following (closed-form) equation:

$$\mathbf{v}_\theta = \cos\theta \frac{\mathbf{U}^{\dagger}_{\bullet 1}\mathbf{m}^{\dagger}\mathbf{m}}{\|\mathbf{U}^{\dagger}_{\bullet 1}\mathbf{m}^{\dagger}\mathbf{m}\|_2} + \sin\theta \frac{\mathbf{m}^{*}\left(\mathbf{m}^{T}\mathbf{U}^{\dagger}_{\bullet 1} - \mathbf{U}^{*}_{\bullet 1}\mathbf{m}\right)}{\|\mathbf{m}^{*}\left(\mathbf{m}^{T}\mathbf{U}^{\dagger}_{\bullet 1} - \mathbf{U}^{*}_{\bullet 1}\mathbf{m}\right)\|_2} \tag{10}$$

Moreover, in the context of this work, it is also proven that in MISO systems the average mutual information for the beamforming scenario can be calculated by an infinite-series, which converges fast (only a few tens of terms are required) to the corresponding value calculated by using Monte Carlo integration with thousands of channel samples [7]-[8]:

$$\mathcal{I}_{\text{bf,avg}}(\theta) = \mathcal{I}_{\text{bf,avg}}(\text{SNR}, \mathbf{v}_\theta) = (ln2)^{-1}\, exp\left(\frac{1-\text{SNR}\,|\boldsymbol{\mu}\mathbf{v}^{\dagger}|^2}{\text{SNR}\,\mathbf{v}\mathbf{R}\mathbf{v}^{\dagger}}\right) \times \sum_{n=0}^{\infty}\left[\frac{1}{n!}\left(\frac{|\boldsymbol{\mu}\mathbf{v}^{\dagger}|^2}{\mathbf{v}\mathbf{R}\mathbf{v}^{\dagger}}\right)^{n} \sum_{k=0}^{n}\left(\frac{1}{\text{SNR}\,\mathbf{v}\mathbf{R}\mathbf{v}^{\dagger}}\right)^{k}\Gamma\left(-k, \frac{1}{\text{SNR}\,\mathbf{v}\mathbf{R}\mathbf{v}^{\dagger}}\right)\right]\Bigg|_{\mathbf{v}=\mathbf{v}_\theta} \tag{11}$$

Using the infinite-series (11) in Theorem 1 and 2 (see equations (1)-(2)) the 1-D method for the calculation of the optimum beamformer in MISO systems can be further simplified and hence, the relative computational complexity is further reduced.

2.2 MIMO systems

We consider a MIMO $N \times M$ flat fading channel with a complex Gaussian distribution $\mathbf{H} \sim \mathcal{N}(\text{vec}(\mathbf{H}_m), \mathbf{R})$, with a *rank*-1 channel mean $\mathbf{H}_m \neq \mathbf{0}$ ($\mathbf{H}_m$ represents the LOS component) and covariance $\mathbf{R} = \mathbf{R}_t^T \otimes \mathbf{R}_r \neq \mathbf{I}_{MN}$, with $\mathbf{R}_t/\mathbf{R}_r$ the channel transmit/receive covariance matrices respectively. Assuming perfect CSI at the receiver and CDIT, it can be proven [9] that the rate-optimum beamforming transmission for the channel model under consideration (spatially correlated or uncorrelated with a non-unit covariance matrix MIMO Rician flat fading channel) is the solution of an 1-D optimization problem, which is expressed using Theorem 1 with the following modification: the normalized channel mean vector in the MISO case, which was denoted in Theorem 1 as $\mathbf{m}$, is replaced by the complex conjugate transpose of the right singular vector of $\mathbf{H}_m$, denoted as $\mathbf{q} \in \mathbb{C}^{1\times N}$, in the MIMO case[1].

In the same manner, for MIMO $N \times M$ flat fading channels with $N = 2$ or $N \geqslant 3$ and $rank\{\mathbf{R}_t\} \leqslant 2$, $\mathbf{v}_\theta$ is expressed using equation (10) (Theorem 2) and replacing $\mathbf{m}$ with $\mathbf{q}$ [9].

2.3 Results for the computational complexity of the 1-D method

The computational complexity of the proposed 1-D method - expressed as the runtime (in seconds) per iteration - is presented via simulations with respect to:

a. The number of channel samples which where used for the calculation of the ergodic beamforming capacity with the Monte Carlo method.

b. The number of transmit antenna elements (N).

The aforementioned complexity is compared with the corresponding complexity of the following multi-dimensional algorithms, which can also be employed in order to calculate the optimum beamformer:

a. An interior-point algorithm with logarithmic barrier function, for MISO and MIMO systems [10].

b. An iterative asymptotic (and hence, sub-optimum) approach, for MISO systems. This algorithm was recently developed in [11] and calculates the optimum beamformer (only) when the optimality of beamforming condition is satisfied.

The simulations were for Uniform Linear Arrays (ULAs) under the two-path delay spread correlation model, which was studied by Winters in [12].

Results are presented in Figure 3 for various scenarios[2] and demonstrate that the proposed 1-D method has significantly lower computational complexity, as follows:

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[1]Since $rank(\mathbf{H}_m) = 1$, it is $\mathbf{H}_m = \mu\mathbf{pq}$, with μ the unique eigenvalue of $\mathbf{H}_m$ and $\mathbf{p} \in \mathbb{C}^{M\times 1}, \mathbf{q}^\dagger \in \mathbb{C}^{N\times 1}$ its left and right singular vectors, respectively.

[2]The exact parameters of these scenarios can be found in [8]-[9].

a. For the simulated scenarios related to MISO systems, the runtime of the 1-D method is on average 5 to 7 times faster than the interior-point method and 2 to 10 times faster than the asymptotic approach.

b. For the simulated scenarios related to MIMO systems, the runtime of the 1-D algorithm is approximately 8.5 times faster than the interior-point method.

3 Results for the optimality of beamforming condition

As referred to in paragraph 1.2, the optimum beamformer achieves ergodic capacity when a necessary and sufficient optimality of beamforming condition is satisfied. This condition was proven in [6] an is expressed by the following inequality:

$$\lambda_{max}\Big((\mathbf{I}_N - \mathbf{v}_{\text{opt}}^{\dagger}\mathbf{v}_{\text{opt}})\mathbf{K}(\mathbf{I}_N - \mathbf{v}_{\text{opt}}^{\dagger}\mathbf{v}_{\text{opt}})^{\dagger}\Big) \leqslant \mathbf{v}_{\text{opt}}\mathbf{K}\mathbf{v}_{\text{opt}}^{\dagger} \tag{12}$$

where $\lambda_{max}(\cdot)$ stands for the maximum eigenvalue and $\mathbf{K} \in \mathbb{H}_{+}^{N}$ is expressed as:

$$\mathbf{K} = \mathcal{E}_{\mathbf{H}}\Big[\mathbf{H}^{\dagger}(\mathbf{I}_M + \text{SNR}\mathbf{H}\mathbf{v}_{\text{opt}}^{\dagger}\mathbf{v}_{\text{opt}}\mathbf{H}^{\dagger})^{-1}\mathbf{H}\Big] \tag{13}$$

Condition (12) is studied in this work for correlated MIMO Rician flat fading channels (assuming perfect CSI at the receiver and CDIT), i.e. the combined CMI-CCI model [13]. The parameters that affect condition (12) - and hence, the *optimality region*, which is defined as the set of channel distribution parameters that satisfy condition (12)- are studied via simulations with the help of a probabilistic approach, leading to important observations:

Observation 1. Beamforming becomes the rate-optimum strategy as the SNR decreases.

Observation 2. Beamforming becomes the rate-optimum strategy as the singular value of $\mathbf{H}_m$, μ, increases.

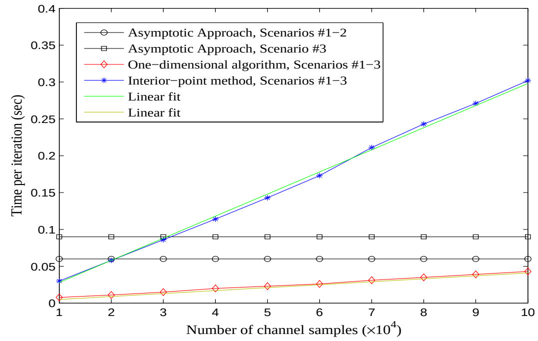
Observation 3. Beamforming becomes the rate-optimum strategy as the channel variance β decreases.

Observation 4. Beamforming becomes the rate-optimum strategy as ϕ (see (4)) decreases.

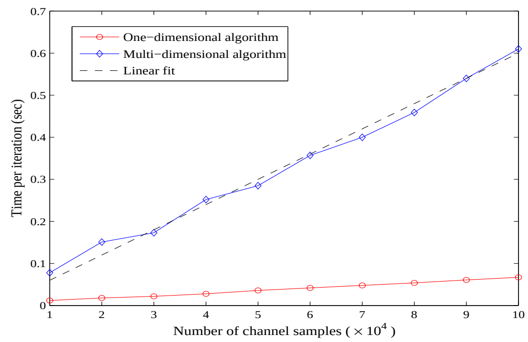
Observation 5. Relatively low β values lead to abrupt increase of the optimality region for relatively high values of the transmit antenna correlation coefficient ρ_t. Moreover, in the low-ρ_t regime, the optimality region seems to be less "sensitive" (i.e. is less affected) to an increase of the SNR, β and ϕ, compared to the high-ρ_t regime.

Observation 6. Beamforming becomes the rate-optimum strategy as the number of receive antenna elements (M) decreases.

The results show that the CDIT model under consideration incorporates the basic characteristics of the uncorrelated MIMO Rician model (addressed with observations 1-3 and 6), however, the model also reveals new characteristics presented for the first time in this work (addressed with observations 4 and 5).



(a)



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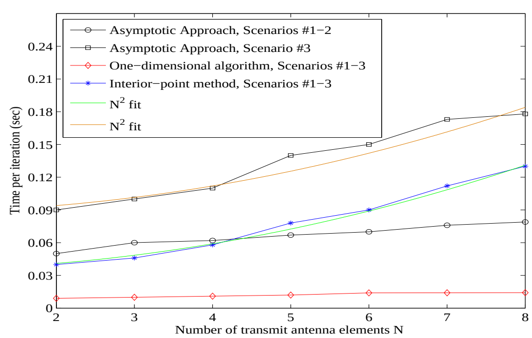

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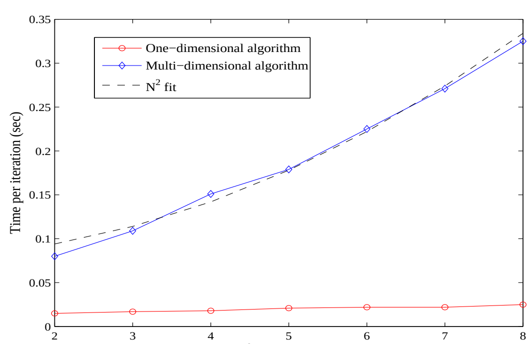

(d)

Figure 3: Runtime vs. the number of channel samples for a MISO 4×1/MIMO 4×4 system (a)/(b), and N with 2×10^4 channel samples and $M = 1/M = 4$ (c)/(d).

Observations 1-5 are visualized in Figure 4. This figure shows a set of curves on the $\mu - \rho_t$ plane, for different β values and $\phi = 35^o/65^o$, SNR $= 0/3$dB and Rx Angular Spread $\Delta_r = 68^o$. Each curve represents a bound: any $\{\mu, \rho_t\}$ point above this bound - i.e. in the region indicated with $Pr_{bf} = 1$ - corresponds to an operational scenario where the optimum beamformer achieves ergodic capacity, i.e. (12) is statistically always satisfied. The $\mu - \rho_t$ region where $Pr_{bf} = 1$ is referred to as the "optimality region".

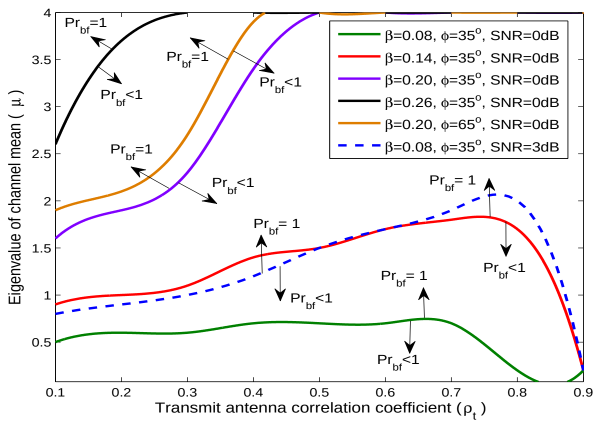

Figure 4: Optimality region $\mu - \rho_t$, for a MIMO 4×4 system and $\{$SNR $= 0/3$dB, $\phi = 35^o/65^o$, $\Delta_r = 68^o\}$.

4 Results using the 3GPP MIMO channel model

In the last part of this work the 3GPP MIMO channel model [14] is employed in order to study the performance of the optimum beamformer with respect to condition (12), using a probabilistic analysis and assuming perfect CSI at the receiver and CDIT. Results are derived for the following cases:

a. Urban micro-cellular environment with LOS component. The long term statistics of this environment simulates best a correlated MIMO Rician flat fading channel and hence, the long term combined CMI-CCI model can be employed [15]. In this case, the optimum beamformer achieves ergodic capacity with probability 0.9 for a wide SNR range.

b. Suburban and urban macro/micro-cellular environments [16]. The long term statistics of these environments simulate a MIMO Rayleigh flat fading channel, where both the CMI/CCI model can be employed, as a short/long term model, respectively. The analysis showed that in both CDIT models the optimum beamformer achieves ergodic capacity with a probability $\geqslant$0.45, in all operational environments and for a wide SNR range.

5 Conclusions

In this doctoral thesis the multi-dimensional and computationally complex optimization problem for the calculation of the rate-optimum beamforming transmission in correlated MISO/MIMO Rician flat fading channels (combined CMI-CCI model) is transformed into a simple 1-D optimization problem, which can be subsequently solved using standard 1-D search algorithms, reducing system's complexity. The reduced complexity can be exploited to either reduce cost by using devices with lower processing power or in order to: (a) operate in environments with smaller coherence time, proportional to the relative processing gain, and hence, support operational scenarios with higher mobility, (i.e. higher speeds, proportional to the relative processing gain), (b) increase the available processing power required by the system for other supplementary techniques. Moreover, the optimality of beamforming condition is studied via simulations for the combined CMI-CCI using a probabilistic analysis, and the optimality region is plotted for different values of the channel distribution parameters and the SNR. Generally, the knowledge of the optimality region can be valuable during the system design and deployment phases: if information for the targeted operational scenarios/channels is available, it can be used to produce the optimality regions and hence, decide if optimum beamforming can be employed as the main transmission strategy, which ultimately leads to reducing the system's complexity and cost. Results demonstrate that there is a wide range of operational scenarios and SNR values where the optimum beamformer achieves ergodic capacity or shows a relatively high (or near-optimum) performance, a fact that justifies the significance of the proposed solutions and the contribution of this work.

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